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Methodology for decomposition analysis of future risk of hunger

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将来の飢餓リスクに関する要因分解分析手法

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Abstract

This document summarizes the methodology for decomposition analysis of future risk of hunger conducted by Hasegawa et al. (in prep) “Scenario of risk of hunger for the 21st century using Shared Socioeconomic Pathways”. The paper cites this document as supporting information. The first chapter describes a methodology for calculation of population at risk of hunger and the second chapter describes a methodology for decomposition analysis of future risk of hunger.

1. Methodology for calculation of population at risk of hunger

Undernourishment or hunger is defined as a clear and narrow concept as a state of energy (calorie) deprivation lasting over a year and do not mean short-lived effects of temporary crises[1]. Furthermore, they do not capture inadequate intake of other essential nutrients[1].

Population at risk of huger is described as a multiple of population and proportion of the population at risk of hunger as (Eq.1).

$$Risk_t = POP_t \cdot PoU_t \qquad \text{(Eq.1)}$$

where,

t : year

$Risk_t$: population at risk of hunger in year t [person]

POP_t : population in year t [person]

PoU_t : proportion of the population at risk of hunger in year t [−]

According to FAO methodology[2], proportion of the population at risk of hunger is defined as (Eq.2) to (Eq.4). In the methodology, the proportion is calculated by using three parameters; mean food calorie intake per person per day (*cal*), mean minimum dietary energy requirement (*M*), and Coefficient of Variation of the inter-national food distribution of dietary energy consumption (*CV*). The inter-national food distribution is assumed to show a standard normal cumulative distribution. Proportion of population under the mean minimum dietary energy requirement (*M*) is defined as a proportion of the population at risk of hunger. The standard normal cumulative distribution has two parameters, mean μ_t and variation σ_t as (Eq.2). The two parameters $\mu_{t,}$ σ_t can be represented by using the mean food calorie intake per person per day (*cal*) and coefficient of variation of the inter-national distribution of dietary energy consumption (*CV*) as (Eq.3), (Eq.4).

The weight-based consumption of food goods calculated by the CGE model is converted into calorie-based consumption by using conversion factors for each commodity and then used as mean food calorie intake per person per day (*cal*). Calories per 100 g[3] are weighted on the basis of production data[4] in the base year and aggregated to the commodity classification to obtain the conversion factors. In this process, only the edible parts of commodities are considered for food consumption by using edible parts ratios[3]. *CV* is an indicator of food security objected by household survey by FAO. It has a range of 0 to 1. FAO country data of CV is weighted on the basis of population data in the base year and aggregated to regional classification to obtain the CV of aggregated regions.

$$PoU_t = \Phi\left(\left. \log M_t - \mu\left(cal_t, \sigma_t\right) \right/ \sigma_t \right) \qquad \text{(Eq.2)}$$

$$\mu\left(cal_t, \sigma_t\right) = \log_e cal_t - \sigma_t^2 / 2 \qquad \text{(Eq.3)}$$

$$\sigma_t = \left[\log_e \left(CV^2 + 1 \right) \right]^{0.5} \qquad \text{(Eq.4)}$$

where,

M_t : mean minimum dietary energy requirement in year t

CV_t : coefficient of variation of the inter-national distribution of dietary energy consumption in year t

Φ : standard normal cumulative distribution

cal_t : mean food calorie intake per person per day in year t

Mean minimum dietary energy requirement (M) is calculated for each year and country by using mean minimum dietary energy requirement in the base year at the country level[4], adjustment coefficient of minimum energy requirements per person of different age and sex groups (Table 1)[5] and population of each age and sex group in each year [6] as (Eq.5) and (Eq.6).

$$M_t = Mbase \cdot \frac{MER_t}{MERbase} \quad \text{(Eq.5)}$$

$$MER_t = \frac{\sum_{i,j} RMER_{i,j} \cdot Pclass_{i,j,t}}{\sum_{i,j} Pclass_{i,j,t}} \quad \text{(Eq.6)}$$

where,
i: age group;
j: sex;
$Mbase$: mean minimum dietary energy requirement per person in base year;
MER_t: Mean adjustment coefficient of minimum energy requirements per person in year t;
MER_{base}: Mean adjustment coefficient of minimum energy requirements per person in base year;
$RMER_{i,j}$: Adjustment coefficient of minimum energy requirements per person of age i and sex j;
$Pclass_{i,j,t}$: population of age i and sex j in year t.

Table 1 Adjustment coefficient of minimum energy requirements per person of different age and sex groups ($RMER_{i,j}$)(average = 1.0)

Age group (years)	Male	Female
0-4	0.46	0.59
5-9	0.75	0.97
10-14	0.97	1.13
15-19	1.02	1.05
20-39	1.00	1.00
40-40	0.95	0.95
50-59	0.90	0.90
60-69	0.80	0.80
70+	0.70	0.70

Note: this table is based on Table 26 in FAO/WHO(1973)

2. Methodology for decomposition analysis of future risk of hunger

Expansion of (Eq.2) shows that change in population at risk of hunger can be decomposed into several factors. A calculation way of effect of each factor is described below.

Partial differentiation of (Eq.2) by σ, cal gives (Eq.7).

$$dPoU_t = \frac{\partial \Phi_t}{\partial \sigma_t} d\sigma_t + \frac{\partial \Phi_t}{\partial cal_t} dcal_t \quad \text{(Eq.7)}$$

Differentiation of (Eq.1) by *Risk* gives (Eq.8).

$$
\begin{aligned}
dRisk_t &= \frac{\partial Risk_t}{\partial POP_t} dPOP_t + \frac{\partial Risk_t}{\partial PoU_t} dPoU_t \\
&= PoU_t \cdot dPOP_t + POP_t \cdot dPoU_t
\end{aligned}
$$

$$
\begin{aligned}
\frac{dRisk_t}{Risk_t} &= \frac{PoU_t \cdot dPOP_t + POP_t \cdot dPoU_t}{PoU_t \cdot POP_t} \\
&= \frac{dPOP_t}{POP_t} + \frac{dPoU_t}{PoU_t} \\
&= \frac{dPOP_t}{POP_t} + \frac{1}{PoU_t} \cdot \left\{ \frac{\partial \Phi_t}{\partial \sigma_t} d\sigma_t + \frac{\partial \Phi_t}{\partial cal_t} dcal_t \right\} + \varepsilon_t \quad (by\ Eq.7) \\
&= IPOP_t + \frac{1}{PoU_t} \cdot \left(ICV_t + ICAL_t \right) + \varepsilon_t \qquad \text{(Eq.8)}
\end{aligned}
$$

where,

$IPOP_t$: change in population at risk of hunger caused by change in population in year t [−]

ICV_t : change in population at risk of hunger caused by change in CV in year t [−]

$ICAL_t$: change in population at risk of hunger caused by change in food calorie intake in year t [−]

ε_t : error of year t

Multiplying *Risk* to both sides of (Eq.8), gives (Eq.9).

$$
\begin{aligned}
dRisk_t &= Risk_t \cdot IPOP_t + \frac{Risk_t}{PoU_t} \cdot ICV_t + \frac{Risk_t}{PoU_t} \cdot ICAL_t + \varepsilon_t \\
&= FaPOP_t + FaCV_t + FaCAL_t + \varepsilon_t \qquad \text{(Eq.9)}
\end{aligned}
$$

with

$$FaPOP_t = Risk_t \cdot IPOP_t$$

$$FaCV_t = \frac{Risk_t}{PoU_t} \cdot ICV_t$$

$$FaCAL_t = \frac{Risk_t}{PoU_t} \cdot ICAL_t$$

where

$dRisk_t$: annual change in population at risk of hunger in year t [person/year]

$FaPOP_t$: annual change in population at risk of hunger caused by change in population in year t [person/year]

$FaCV_t$: annual change in population at risk of hunger caused by change in CV in year t [person/year]

$FaCAL_t$: annual change in population at risk of hunger caused by change in food calorie intake in year t [person/year]

(Eq.9) represents that change in population at risk of hunger in year t can be decomposed into change in three factors: population, variation of food distribution and per-capita food calorie intake.

Accumulation of (Eq.9) from base year to year t, gives (Eq.10).

$$
\begin{aligned}
Risk_t &= Risk_{tbase} + \left(dRisk_{tbase+1} + \cdots + dRisk_t \right) \\
&= Risk_{tbase} + \sum_{tbase < t' \le t} dRisk_{t'} \qquad \text{(Eq.10)}
\end{aligned}
$$

Thus, cumulative change in population at risk of hunger from the base year to year *t* is described as a sum of cumulative change in the three factors as (Eq.11).

$$Risk_t - Risk_{tbase} = \sum_{tbase<t'\le t} dRisk_{t'}$$
$$= \sum_{tbase<t'\le t} \left(FaPOP_{t'} + FaCV_{t'} + FaCAL_{t'}\right) + \varepsilon_t$$
$$= FPOP_t + FCV_t + FCAL_t + \varepsilon_t \qquad \text{(Eq.11)}$$

with

$$FPOP_t = \sum_{tbase<t'\le t} \left(FaPOP_{t'}\right)$$
$$FCV_t = \sum_{tbase<t'\le t} \left(FaCV_{t'}\right)$$
$$FCAL_t = \sum_{tbase<t'\le t} \left(FaCAL_{t'}\right)$$

where

$FPOP_t$:cumulative change in population at risk of hunger caused by change in population from base year to year t [person]
FCV_t :cumulative change in population at risk of hunger caused by change in CV from base year to year t [person]
$FCAL_t$:cumulative change in population at risk of hunger caused by change in food calorie intake from base year to year t [person]

Using the above formula, it is possible to describe cumulative change in population at risk of hunger from base year to year *t* as (Eq.12) to (Eq.14).

$$FPOP_t = \sum_{tbase<t'\le t} \left(FaPOP_{t'}\right) = \sum_{tbase<t'\le t} \left(Risk_{t'} \cdot IPOP_{t'}\right) \qquad \text{(Eq.12)}$$

$$FCV_t = \sum_{tbase<t'\le t} \left(FaCV_{t'}\right) = \sum_{tbase<t'\le t} \left(\frac{Risk_{t'}}{PoU_{t'}} \cdot ICV_{t'}\right) \qquad \text{(Eq.13)}$$

$$FCAL_t = \sum_{tbase<t'\le t} \left(FaCAL_{t'}\right) = \sum_{tbase<t'\le t} \left(\frac{Risk_{t'}}{PoU_{t'}} \cdot ICAL_{t'}\right) \qquad \text{(Eq.14)}$$

In order to calculate *FPOP, FCV* and *FCAL,* a way of calculating *IPOP, ICV* and *ICAL* is described below.

First, effects of change in population on change in population at risk of hunger (*IPOP*) can be calculated as (Eq.15).

$$IPOP_t = \frac{\partial POP_t}{POP_t} = \frac{\left(POP_{t+1} - POP_t\right)}{POP_t} \qquad \text{(Eq.15)}$$

Second, effects of change in variation of food distribution on change in population at risk of hunger (*ICV*) can be calculated as (Eq.16) by assuming approximation.

$$ICV_t = \frac{\partial \Phi_t}{\partial \sigma_t} d\sigma_t \approx \partial \Phi_t = \Phi_t(\sigma_{t+1}) - \Phi_t(\sigma_t)$$
$$= \Phi\left(\frac{\log(M_t) - \mu(cal_t, \sigma_{t+1})}{\sigma_{t+1}}\right) - \Phi\left(\frac{\log(M_t) - \mu(cal_t, \sigma_t)}{\sigma_t}\right) \quad \text{(Eq.16)}$$

Finally, effects of change in per-capita food calorie intake (*ICAL*) is calculated as follows. Since per-capita food calorie intake is calculated from income, income elasticity, price and price elasticity as (Eq.17), *ICAL* can be decomposed into effects of change in income (*IY*) and those in price (*IPC*) as (Eq.18).

$$cal_t = \sum_i cali_{t,i} = \sum_i \left(Y_t^{\alpha_{t,i}} \cdot P_{t,i}^{\ \beta_{t,i}} \cdot \gamma_{t,i} \right) \quad \text{(Eq.17)}$$

$$ICAL_t = \frac{\partial \Phi_t}{\partial cal_t} dcal = \frac{\partial \Phi_t}{\partial Y_t} dY_t + \sum_i \frac{\partial \Phi_t}{\partial P_{t,i}} dP_{t,i} = IY_t + \sum_i IPC_{t,i}, \quad \text{(Eq.18)}$$

with

$$IY_t = \frac{\partial \Phi_t}{\partial Y_t} dY_t$$

$$IPC_{t,i} = \frac{\partial \Phi_t}{\partial P_{t,i}} dP_{t,i}$$

where

$cali_{t,i}$: mean calorie intake per person per day for commodity i, in year t

Y_t : income in year t

$P_{i,t}$: price of commodity i in year t

$\alpha_{t,i}$: income elasticity of demand for commodity i, year t

$\beta_{i,t}$: price elasticity of demand for commodity i, year t

$\gamma_{i,t}$: constant term for commodity i, year t

IY_t : change in population at risk of hunger caused by change in income in year t [−]

$IPC_{t,i}$: change in population at risk of hunger caused by change in price of commodity i, in year t [−]

First, the effects of change in income (*IY*) can be described as (Eq.19). Since income elasticity is different across commodity *i*, the effects of change in income are decomposed into the effects of change in income to change in calorie intake from each commodity (*IYC*).

$$IY_t = \frac{\partial \Phi_t}{\partial Y_t} dY_t = \frac{\partial \Phi_t}{\partial cal_t} \cdot \frac{\partial cal_t}{\partial Y_t} dY_t$$
$$= \frac{\partial \Phi_t}{\partial cal_t} \cdot dY_t \cdot \sum_i \left(\frac{\partial cali_{t,i}}{\partial Y_t} \right) = \frac{\partial \Phi_t}{\partial cal_t} \cdot dY_t \cdot \sum_i IYC_{t,i} \quad \text{(Eq.19)}$$

with

$$IYC_{t,i} = \frac{\partial cali_{t,i}}{\partial Y_t}$$

where

$IYC_{t,i}$: change in mean calorie intake per person per day from commodity i caused by change in income in year t [−]

In order to calculate the IY by (Eq.19), $\frac{\partial \Phi_t}{\partial cal_t} \cdot dY_t$ and $IYC_{t,i}$ are described as (Eq.20) and (Eq.21).

$$\frac{\partial \Phi_t}{\partial cal_t} \cdot dY_t = \frac{\Phi(cal_{t+1}) - \Phi(cal_t)}{cal_{t+1} - cal_t} \cdot (Y_{t+1} - Y_t) \quad \text{(Eq.20)}$$

With the definition of $cali_{t,i} = Y_t^{\alpha_{t,i}} \cdot P_{t,i}^{\ \beta_{t,i}} \cdot \gamma_{t,i}$,

$$IYC_{t,i} = \frac{\partial cali_{t,i}}{\partial Y_t} = \alpha_t \cdot Y_t^{\alpha_{t,i}-1} \cdot P_{t,i}^{\ \beta_{t,i}} \cdot \gamma_{t,i} = \alpha_t \cdot \frac{cali_{t,i}}{Y_t} \quad \text{(Eq.21)}$$

By assigning (Eq.20), (Eq.21) to (Eq.19), IYt is calculated as (Eq.22).

$$\begin{aligned} IY_t &= \frac{\Phi(cal_{t+1}) - \Phi(cal_t)}{cal_{t+1} - cal_t} \cdot (Y_{t+1} - Y_t) \cdot \sum_i \left(\alpha_t \cdot \frac{cali_{t,i}}{Y_t} \right) \\ &= \frac{\Phi\left(\frac{\log(M_{t+0.5}) - \mu(cal_{t+1}, \sigma_{t+0.5})}{\sigma_{t+0.5}} \right) - \Phi\left(\frac{\log(M_{t+0.5}) - \mu(cal_t, \sigma_{t+0.5})}{\sigma_{t+0.5}} \right)}{cal_{t+1} - cal_t} \\ &\qquad \cdot (Y_{t+1} - Y_t) \cdot \sum_i \left(\alpha_{t,i} \cdot \frac{cali_{t,i}}{Y_t} \right) \end{aligned} \quad \text{(Eq.22)}$$

where, $\sigma_{t+0.5} = \frac{\sigma_{t+1} + \sigma_t}{2}, M_{t+0.5} = \frac{M_{t+1} + M_t}{2}$

Second, the effect of price change (IPC) is described as (Eq.23).

$$IPC_{t,i} = \frac{\partial \Phi_t}{\partial P_{t,i}} dP_{t,i} = \frac{\partial \Phi_t}{\partial cali_{t,i}} \cdot \frac{\partial cali_t}{\partial P_{t,i}} dP_{t,i} \quad \text{(Eq.23)}$$

In order to calculate the IPC by (Eq.23), $\frac{\partial \Phi_t}{\partial cali_{t,i}} \cdot dP_{t,i}$ and $\frac{\partial cali_{t,i}}{\partial P_t}$ are described as (Eq.24) and (Eq.25) respectively.

$$\frac{\partial \Phi_t}{\partial cali_{t,i}} \cdot dP_{t,i} = \frac{\Phi(cali_{t+1,i}) - \Phi(cali_{t,i})}{cali_{t+1,i} - cali_{t,i}} \cdot (P_{t+1,i} - P_{t,i}) \quad \text{(Eq.24)}$$

With the definition of $cali_{t,i} = Y_t^{\alpha_{t,i}} \cdot P_{t,i}^{\ \beta_{t,i}} \cdot \gamma_{t,i}$,

$$\frac{\partial cali_{t,i}}{\partial P_t} = \beta_{t,i} \cdot Y_t^{\alpha_{t,i}} \cdot P_{t,i}^{\ \beta_{t,i}-1} \cdot \gamma_{t,i} = \beta_{t,i} \cdot \frac{cali_{t,i}}{P_{t,i}} \quad \text{(Eq.25)}$$

By assigning (Eq.24), (Eq.25) to (Eq.23), IPC is calculated as (Eq.26).

$$IPC_{t,i} = \frac{\Phi\left(cali_{t+1,i}\right) - \Phi\left(cali_{t,i}\right)}{cali_{t+1,i} - cali_{t,i}} \cdot \left(P_{t+1,i} - P_{t,i}\right) \cdot \beta_{t,i} \cdot \frac{cali_{t,i}}{P_{t,i}}$$

$$= \frac{\left[\Phi\left(\frac{\log M_{t+0.5} - \mu i\left(cali_{t+1,i} + \sum_{i',i\neq i'} cali_{t,i'}, \sigma_{t+0.5}\right)}{\sigma_{t+0.5}}\right) - \Phi\left(\frac{\log M_{t+0.5} - \mu\left(\sum_{i} cali_{t,i}, \sigma_{t+0.5}\right)}{\sigma_{t+0.5}}\right)\right]}{cali_{t+1,i} - cali_{t,i}} \cdot \left(P_{t+1,i} - P_{t,i}\right) \cdot \beta_{t,i} \cdot \frac{cali_{t,i}}{P_{t,i}}$$

(Eq.26)

Then, assigning (Eq.22) and (Eq.26) to (Eq.18), gives the effect of change in per-capita food calorie intake (*ICAL*) .

Using the above formula, effects of the three factors on population at risk of hunger are calculated.

First, cumulative change in population at risk of hunger caused by population change (*FPOP*) is calculated by assigning (Eq.15) to (Eq.12).

Second, cumulative change in population at risk of hunger caused by change in variation of food distribution (*FCV*) is calculated by assigning (Eq.16) to (Eq.13).

Third, cumulative change in population at risk of hunger caused by change in per-capita food calorie intale (*FCAL*) is calculated by assigning (Eq.18) to (Eq.14).

Finally, error (ε_t) is calculated as (Eq.27).

$$\varepsilon_t = \left(Risk_t - Risk_{tbase}\right) - \left(FPOP_t + FCV_t + FCAL_t\right) \qquad \text{(Eq.27)}$$

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